Averaging of fast-slow Hamiltonian systems and a thermodynamic interpretation

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Atoms and molecules are on a microscopic scale governed by Hamiltonian dynamics, namely Newton's equations of motion. Their dynamics is thus invariant under time reversal. Yet, the collective macroscopic evolution has a direction of time, usually interpreted as an entropic effect.

How does this transition from reversible Hamiltonian dynamics to irreversible thermodynamic macroscopic evolution happen? In this talk, we consider a hugely simplified situation, namely a fast-slow Hamiltonian system of two particles. This system does not include the many-particle aspect one usually associated with thermodynamics, but captures the most fundamental aspect of thermodynamic systems, namely a separation of scales. The talk will first revisit work of Bornemann, which derives an effective macroscopic evolution of Hamiltonian type. We will then study a higher order expansion, which can be derived rigorously via two-scale convergence. Finally, a thermodynamic interpretation of the resulting system is given, where entropic effects appear as higher-order corrections to energetic ones.

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