Crystallization results for pairwise interaction energies in two dimensions

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I will present some recent crystallization results concerning pairwise interaction energies. First, I will focus on minimizers and quasi-minimizers of the so-called Heitmann-Radin sticky disc functional. I will show that, renormalizing the Heitmann-Radin potential by subtracting the minimal energy per particle (i.e., the kissing number), configurations scaling like a perimeter converge to a polycrystal. More precisely, the empirical measure converges - as the number of particles diverges - to a set of finite perimeter, and a microscopic variable, representing the orientation of the underlying lattice, converges to a locally constant function. Such a compactness result is accompanied by a Gamma-convergence result in the case that the limit configuration is a single crystal, namely it has a constant orientation.

In the second part of the talk, I will consider a new pairwise interaction potential inspired by the Heitmann-Radin one for which I show a crystallization result in the square lattice in the thermodynamic limit. Time permitting, I will show how such a crystallization result can be extended to smooth and long range potentials satisfying suitable growth and convexity assumptions. Finally, I will illustrate some open problems.