Derivation of linear elasticity from atomistic energies with multiple wells

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A rigorous derivation of linear elastic theories from non linear elasticity has been provided in terms of Γ-convergence for both continuum and atomistic models mainly in the case of single well potentials (see for example [3], [2], [4]). On the other hand, energies with multiple wells naturally arise in many models, as for example in the gradient theory of solid-solid phase transitions. In the recent paper [1] it has been shown that linear elasticity can be rigorously derived from multi-well energies by adding a singular higher order term which penalizes the transitions between the wells and turns out to be necessary in order to recover good compactness properties of minimizing sequences of displacement fields. In this talk I will present a recent result in collaboration with G. Lazzaroni and M. Palombaro for the derivation of linear elasticity from a general class of atomistic energies with multiple wells for crystalline materials, showing that the role of the singular term in the continuum model in penalizing jumps from one well to another is played in this setting by interactions beyond nearest neighbours.

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